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MA637 Numerical Analysis & Computing

Winter Semester 24/25

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Assigment 2

Assignment and programming exercises must be completed in groups of 4 members and submitted via Google Classroom by 27.01.2025 at 23:59. If the assignment is not prepared using LaTeX, a clear scanned copy of the handwritten work must be uploaded. Ensure that the names and enrollment numbers of all group members are clearly written on the submission. Late submissions will not be accepted.

Question 1: [**Newton Basis Functions**]

Let $\{x_0, x_1, \dots, x_n\}$ be distinct points. We denote the Newton basis functions by ω_i, that is given by

$$\omega_i = \prod_{k=0}^{i-1} (x - x_k)$$

for $i \geq 1$ and $\omega_0(x) = 1$. Show that $\{\omega_i(x)\}_{i=0}^n$ are linearly independent.

Question 2: [**Newton Divided Difference**]

Let $\{x_0, x_1, x_2\}$ be distinct points. The second degree Newton interpolating polynomial is given by

$$p_2^{\mathbb{N}}(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1),$$

for some constants $\{a_i\}_{i=0}^2$ such that $p_2^{\mathbb{N}}(x_i) = f_i$ for $i = 0, 1, 2$. Derive the expression

$$p_2^{\mathbb{N}}(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1),$$

by computing the coefficients a_i for $i = 0, 1, 2$ directly.

Question 3: [**Programming Exercise**]

The goal of this exercise is to understand polynomial interpolation using the Lagrange Interpolation algorithm and analyze its accuracy for different dataset sizes. Create a data set of points $\{(x_i, f_i)\}_{i=0}^n$ using the functions

$$f_1(x) = \cos(x), \quad \text{and} \quad f_2(x) = \frac{1}{1 + 25x^2},$$

in the interval $[-1, 1]$.

a) Write a code for polynomial interpolation using the Lagrange Interpolation algorithm.

b) Use $n = 5, 25, 50$.

c) Evaluate the interpolated polynomial at $x = 0$ and $x = 0.95$.

i) Compute the error ($\texttt{Error} = |\texttt{Interpolated Value} - \texttt{Exact Value}|$) and comment for what values of n in **b)** do you observe the least error and give justification.

ii) Compute the time taken to evaluate the polynomial at $x = 0$ for $f_2(x)$ and all n mentioned in **b)**.

Hint: In this programming exercise, you would require the following programming concepts:

a) `for` loops.

b) `if-else` structure.

c) Function declaration using `def`.

d) `time` library.

- **i)** `time`: To know the current time of the system.

e) `numpy` library:

- **i)** `linspace`: For creating equidistant points.
- **ii)** `size`: To get the number of elements in the numpy array.
- **iii)** `cos`: To get the cosine function.
- **iv)** `abs`: To get the absolute value.

Question 4: [**Programming Exercise**]

The goal of this exercise is to understand polynomial interpolation using the Newton DD interpolation algorithm and analyze its accuracy for different dataset sizes. Create a data set of points $\{(x_i, f_i)\}_{i=0}^{n}$ using the functions

$$f_1(x) = \cos(x), \quad \text{and} \quad f_2(x) = \frac{1}{1 + 25x^2},$$

in the interval $[-1, 1]$.

a) Write a code for polynomial interpolation using the Newton DD Interpolation algorithm.

b) Use $n = 5, 25, 50$.

c) Evaluate the interpolated polynomial at $x = 0$ and $x = 0.95$.

- **i)** Compute the error ($\texttt{Error} = |\texttt{Interpolated Value} - \texttt{Exact Value}|$) and comment for what values of n in **b)** do you observe the least error.
- **ii)** Compute the time taken to evaluate the polynomial at $x = 0$ for $f_2(x)$ and all the n mentioned in **b)**.
- **iii)** Based on the results in **ii)** and Prob. 3 **c), ii)** what can you infer? Give explanation of the behaviour.

Hint: In this programming exercise, you would require the same concepts as in the previous exercise.