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MA637 Numerical Analysis & Computing

Winter Semester 24/25

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Assigment 3

Assignment and programming exercises must be completed in groups of 4 members and submitted via Google Classroom by 03.02.2025 at 23:59. If the assignment is not prepared using LaTeX, a clear scanned copy of the handwritten work must be uploaded. Ensure that the names and enrollment numbers of all group members are clearly written on the submission. Late submissions will not be accepted.

Question 1: [Hermite Interpolation]

Let $f : [-1, 3] \to \mathbb{R}$ be determined by $f(x) = |1 - x|$.

a) Determine the Hermite interpolation for $x_0 = 0$ and $x_1 = 2$ with function values $f(x_i)$ and $f'(x_i)$ for $i = 0, 1$.

b) Repeat the above task but for $x_0 = 0$ and $x_1 = 1$. For $x_1 = 1$ take $f'(x_1) = 0$ during calculations.

c) Sketch $f(x)$ and the polynomials $p_3^{\mathbb{H}}(x)$ obtained from **a)** and **b)**. Which approximation is better?

Hint: You can use online graphing tools such as DESMOS to plot the graphs in **c)**.

Question 2: [Spline Interpolation]

Let $I := [0, 2\pi] \subset \mathbb{R}$ and $f : I \to \mathbb{R}$ with

x_i	f_i	$f_i^{(1)}$	$f_i^{(2)}$
0	0	1	0
$\pi/2$	1	0	-1
$3\pi/2$	-1	0	1

a) What is the maximum and minimum degree of piecewise interpolation that can be found from the following data? (*Hint:* You can omit some data to get a lower degree).

b) Determine a piecewise quadratic spline using a subset of the data set given above. Which x_i is best suited for this and why?

Question 3: [Trigonometric Interpolation]

Let $f(x) : [0, 2\pi] \to \mathbb{R}$ be a function approximated using trigonometric polynomials say

$$p_n^{\mathbb{T}}(x) = a_0 + \sum_{j=1}^{n} \Big(a_j \cos(jx) + b_j \sin(jx) \Big) .$$

If $|a_n| + |b_n| \neq 0$, then this is called as trigonometric polynomial of degree n.

a) Show that the above polynomial can be re-written in the form:

$$p_n^{\mathbb{T}}(x) = \sum_{j=-n}^{n} c_j e^{ijx},$$

where i stands for $\sqrt{-1}$.

b) Express $\{c_j\}_{j=-n}^{n}$ in the terms of $\{a_j\}_{j=0}^{n}$ and $\{b_j\}_{j=1}^{n}$.

c) If we have $(2n+1)$ distinct points say $0 = x_0 < x_1 < \cdots < x_{2n} = 2\pi$ and their function values $f(x_i)$, then what can we say about the existence and uniqueness of this interpolation.

Question 4: [**Programming Exercise**]

The goal of this exercise is to understand polynomial interpolation using the Hermite Interpolation algorithm and analyze its accuracy for different dataset sizes. Create a data set of points $\{(x_i, f_i)\}_{i=0}^{n}$ using the functions

$$f_1(x) = \cos(x), \quad \text{and} \quad f_2(x) = \frac{1}{1+25x^2},$$

in the interval $[-1, 1]$.

a) Write a code for polynomial interpolation using the Hermite Interpolation algorithm.

b) Use $n = 5, 10$.

c) Evaluate the interpolated polynomial at $x = 0$ and $x = 0.95$.

- **i)** Compute the error (`Error` = |`Interpolated Value`−`Exact Value`|) and comment for what values of n in **b)** do you observe the least error and give justification. How are the results different from Assignment 2, Question 3? What is the reason for this behaviour?

Hint: In this programming exercise, you would require the following programming concepts:

a) `for` loops.

b) `if-else` structure.

c) Function declaration using `def`.

d) Calling of functions from different `.py` files.

e) `time` library.

- **i)** `time`: To know the current time of the system.

f) `numpy` library:

- **i)** `linspace`: For creating equidistant points.

ii) `size`: To get the number of elements in the numpy array.

iii) `cos`: To get the cosine function.

iv) `abs`: To get the absolute value.