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MA637 Numerical Analysis & Computing

Winter Semester 24/25

05.02.2025

Assigment 4

Assignment and programming exercises must be completed in groups of 4 members and submitted via Google Classroom by 14.02.2025 at 23:59. If the assignment is not prepared using LaTeX, a clear scanned copy of the handwritten work must be uploaded. Ensure that the names and enrollment numbers of all group members are clearly written on the submission. Late submissions will not be accepted.

Question 1: [**Matrix Elementary Operations**]

Let $\mathbf{Ax} = \mathbf{b}$ be a system of equations such that $\mathbf{A}$ is a $n \times n$ matrix. Show that the elementary matrix operations:

a) $\mathrm{R}_i \mapsto \lambda \mathrm{R}_i$ for $\lambda \in \mathbb{R}$,

b) $\mathrm{R}_i \mapsto \mathrm{R}_i + \lambda \mathrm{R}_j$ for $j = 1, 2, \ldots, n$ and $j \neq i$, and

c) $\mathrm{R}_i \leftrightarrow \mathrm{R}_j$,

for $i = 1, 2, \ldots, n$ do not change the solution of the system. Here R_i denotes the row i.

Question 2: [**Gauss-Jordan Algorithm**]

Consider the Gauss-Jordan Algorithm provided in Algorithm 7. Show that the computational complexity for the multiplications and divisions is

$$\frac{n^3}{2} + n^2 - \frac{n}{2},$$

and for addition and subtraction is

$$\frac{n^3}{2} - \frac{n}{2}.$$

Question 3: [**LU Decomposition**]

Consider the system of equation $\mathbf{Ax} = \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 \\ -4 & 2 & 4 \\ 6 & 3 & 2 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}.$$

Find a lower unit triangular matrix $\mathbf{L}$ and an upper triangular matrix $\mathbf{U}$ such that $\mathbf{A} = \mathbf{LU}$. Using the LU decomposition, solve for $\mathbf{x}$.

Question 4: [**Programming Exercise**]

The goal of this exercise is to understand polynomial interpolation and analyze its accuracy for different system of equations. Consider the system of equation $\mathbf{A}\mathbf{x} = \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 2 & -2 & 3 & -1 \\ 1 & 1 & 1 & 0 \\ 1 & -1 & 4 & 3 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} -8 \\ -20 \\ -2 \\ 4 \end{bmatrix}.$$

a) Write a code for Gaussian elimination with pivoting to solve the above system of equation to get the numerical solution $\mathbf{x}_{\text{N}}$.

b) The known solution to the problem is $\mathbf{x} = \begin{bmatrix} -7 & 3 & 2 & 2 \end{bmatrix}^\top$. Evaluate the error $\|\mathbf{x} - \mathbf{x}_{\text{N}}\|_{\ell^2}$.

c) Consider

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 8 & 10 \\ 3 & 8 & 14 & 16 \\ 4 & 10 & 16 & 20 \end{bmatrix},$$

and the same $\mathbf{b}$. Compute the solution $\mathbf{x}_{\text{N}}$. What message do you get? Can you modify the code in such a way that you get atleast one solution?

Hint: In this programming exercise, you would require the following programming concepts:

a) `for` loops.

b) `if-else` structure.

c) Function declaration using `def`.

d) `break` statement.

e) `numpy` library:

- **i)** `array`: For creating arrays and matrices.
- **ii)** `size`: To get the number of elements in the numpy array.
- **iii)** `zeros`: To create an array with entries as zero.
- **iv)** `linalg.norm`: Find the $\|\cdot\|_{\ell^2}$ norm.