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MA637 Numerical Analysis & Computing

Winter Semester 24/25

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Assigment 7

Assignment and programming exercises must be completed in groups of 4 members and submitted via Google Classroom by 03.04.2025 at 23:59. If the assignment is not prepared using LaTeX, a clear scanned copy of the handwritten work must be uploaded. Ensure that the names and enrollment numbers of all group members are clearly written on the submission. Late submissions will not be accepted.

Question 1: [**RK-2 Methods**]

Consider the initial value problem

$$\begin{aligned} y'(t) &= -y(t) + t + 1, \quad 0 \le t \le 1 \\ y(0) &= 1. \end{aligned}$$

a) Use the mid-point and the modified Euler method to compute y_1^h and y_2^h for $h = 0.5$. Compare the results obtained with both methods.

b) If we use any other value of h in **a)** then we will still get the same result for both the mid-point and the modified Euler method. Why is it so?

Question 2: [**Butcher Tableau**]

Consider the following Butcher Tableau's. Which of the following represent a Runge-Kutta (RK) scheme, and which do not? Justify your answer and give reasoning.

a)

0		
1	1	
	$1/3$	$1/3$

b)

0			
$1/2$	1		
1	-1	1	2
	$1/6$	$2/3$	$1/3$

c)

0				
$1/3$	$1/3$			
$2/3$	0	$2/3$		
1	0	0	1	
$3/4$	0	0	0	1
	$1/8$	$3/8$	$3/8$	$1/8$

d)

0			
$1/2$	$1/2$		
1	-1	2	
	$1/6$	$2/3$	$1/6$

Question 3: [**Stability Function**]
Consider the initial value problem

$$\begin{aligned} y'(t) &= \lambda y(t), \quad t > 0 \\ y(0) &= 1, \end{aligned}$$

with $\lambda \in \mathbb{R}$. Derive the stability function $\varphi(\ \cdot\ )$ for the modified Euler method and plot it's region of stability.

Question 4: [**Programming Exercise**]
This exercise aims to understand the Higher-Order Taylor method and analyze its accuracy. Consider the initial value problem

$$\begin{aligned} y'(t) &= -t^2 + y(t) + 1, \quad 0 \le t \le 2, \\ y(0) &= 0.5. \end{aligned}$$

a) Write a code to approximate the numerical solution $\{y_i^h\}$ using Taylor-Order methods of order 1, 2, and 3.

b) The analytical solution to the above problem is given by

$$y(t) = (t+1)^2 - 0.5e^t.$$

Compute the error at each t_i for $h = 0.1, 0.2$, and 0.4 and all the orders mentioned in **a)**.

c) What do you observe as h decreases and the order increases?

Hint: In this programming exercise, you would require the following programming concepts:

a) `for` loops.

b) `if-else` structure.

c) Function declaration using `def`.

d) `numpy` library:

- **i)** `linspace`: For creating arrays of equal spacing.
- **ii)** `abs`: To get the absolute value.
- **iii)** `exp`: To get the exponential function.