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MA203 Numerical Methods
Winter Semester 2026
19.03.2026

Assigment 2

Tutorial(Problem 4) must be completed individually and submitted via Google Classroom by 26.03.2026 at 23:59. Ensure that the name and enrollment number is clearly written on the submission. Late submissions will not be accepted.

Question 1: [Fixed Point Method]

a) Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where

$$f(x) = x^4 + 2x^2 - x - 3.$$

i) $g_1(x) = \left(3 + x - 2x^2\right)^{1/4}$.

ii) $g_2(x) = \left(\frac{x+3-x^4}{2}\right)^{1/2}$.

iii) $g_3(x) = \left(\frac{x+3}{x^2+2}\right)^{1/2}$.

iv) $g_4(x) = \frac{3x^4+2x^2+3}{4x^3+4x-1}$.

b) Using $p_0 = 1$, compute $p_{n+1} = g(p_n)$ for $n = 0, 1$ for $g_2(x)$ and $g_3(x)$.

Question 2: [Newton Method]

a) Consider a variation of Newton's method in which the derivative is evaluated at a fixed point α_0, i.e.,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(\alpha_0)}, \tag{1}$$

where $f'(\alpha_0) \neq 0$.

Find constants C and s such that

$$e_{n+1} = Ce_n^s,$$

where $e_n = x_n - x^*$ and $f(x^*) = 0$.

b) Using Eq. (1), compute x_2 for $f(x) = x^2 - 6$ with $x_0 = 1$ and $\alpha_0 = 1$. Compare your results with the standard Newton method.

Hint: Use the Taylor expansion of f about x^* in part (a).

Question 3: [Gaussian Elimination]

Given the linear system

$$
\begin{aligned}
x_1 - x_2 + \alpha x_3 &= -2 \\
-x_1 + 2x_2 - \alpha x_3 &= 3 \\
\alpha x_1 + x_2 + x_3 &= 2
\end{aligned}
$$

Find the value of α for which the system has

a) No solution.

b) Infinitely many solutions.

c) Using $\alpha = 2$, compute the unique solution.

Question 4: [Programming]

Write a Python program implementing the fixed point iteration method. Using a stopping criterion

$$|p_n - p_{n-1}| < 10^{-6},$$

and take 100 as maximum number of iterations. Apply the following four iteration formulas to approximate $\sqrt[3]{21}$, starting with $p_0 = 1$.

a) $p_n = \dfrac{20}{21} p_{n-1} + \dfrac{1}{p_{n-1}^2}$.

b) $p_n = p_{n-1} - \dfrac{p_{n-1}^3 - 21}{3p_{n-1}^2}$.

c) $p_n = p_{n-1} - \dfrac{p_{n-1}^4 - 21p_{n-1}}{p_{n-1}^2 - 21}$.

d) $p_n = \left(\dfrac{21}{p_{n-1}} \right)^{1/2}$.

For each method:

a) Determine whether the iteration converges.

b) If it converges, record the number of iterations required.

c) Rank the methods in order of observed speed of convergence.