Dr. A. Jha

MA203 Numerical Methods

Winter Semester 2026

02.04.2026

Assigment 3

Tutorial(Problem 4) must be completed individually and submitted via Google Classroom by 09.04.2026 at 23:59. Ensure that the name and enrollment number is clearly written on the submission. Late submissions will not be accepted.

Question 1: [**Condition Number**]

a) Compute the condition number κ_∞ of the following matrix in the ∞-norm:

$$\mathbf{A} = \begin{bmatrix} 3.9 & 1.6 \\ 6.8 & 2.9 \end{bmatrix}.$$

b) Consider the following system of equations:

$$\begin{aligned} 3.9x_1 + 1.6x_2 &= 5.5, \\ 6.8x_1 + 2.9x_2 &= 9.7. \end{aligned}$$

The exact solution is $\mathbf{x} = (1, 1)$ and an approximate solution is $\tilde{\mathbf{x}} = (0.98, 1.1)$.

Using the result from part (a), compute:

i) $\|\mathbf{x} - \tilde{\mathbf{x}}\|_\infty$.

ii) $\kappa_\infty(\mathbf{A}) \dfrac{\|\mathbf{b} - \mathbf{A}\tilde{\mathbf{x}}\|_\infty}{\|\mathbf{A}\|_\infty}$.

Question 2: [**Iterative Methods**]

Find the first two iterations of the Jacobi and the Gauss–Seidel methods for the following linear system using the initial iterate $\mathbf{x}^{(0)} = \mathbf{0}$:

$$\begin{aligned} 4x_1 + x_2 - x_3 &= 5, \\ -x_1 + 3x_2 + x_3 &= -4, \\ 2x_1 + 2x_2 + 5x_3 &= 1. \end{aligned}$$

Question 3: [**Interpolation**]

Construct the interpolating polynomial for the following data:

x	-0.1	0.0	0.2	0.3
$f(x)$	5.3	2.0	3.19	1.0

using:

a) Newton's Divided Difference method,

b) Lagrange's interpolation formula.

Further:

(a) What is the degree of the polynomial obtained?

(b) Evaluate the polynomial at $x = -0.05$.

(c) If we add the data point $(0.35, 0.9726)$, which method is more convenient? Compute the updated value at $x = -0.05$.

Question 4: [**Programming Exercise**]

The goal of this exercise is to understand polynomial interpolation using the Lagrange Interpolation algorithm and analyze its accuracy for different dataset sizes. Create a data set of points $\{(x_i, f_i)\}_{i=0}^{n}$ using the functions

$$f_1(x) = \cos(x), \quad \text{and} \quad f_2(x) = \frac{1}{1 + 25x^2},$$

in the interval $[-1, 1]$.

a) Write a code for polynomial interpolation using the Lagrange Interpolation algorithm.

b) Use $n = 5, 25, 50$.

c) Evaluate the interpolated polynomial at $x = 0$ and $x = 0.95$. Compute the error ($\texttt{Error} = |\texttt{Interpolated Value} - \texttt{Exact Value}|$) and comment for what values of n in **b)** do you observe the least error and give justification.