



Tutorial(Problem 4) must be completed individually and submitted via Google Classroom by 17.04.2026 at 23:59. Ensure that the name and enrollment number is clearly written on the submission. Late submissions will not be accepted.

Question 1: [Integration]

- a) Find the degree of precision of the quadrature formula

$$\int_{-1}^1 f(x) dx = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right).$$

- b) The quadrature formula

$$\int_{-1}^1 f(x) dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$$

is exact for all polynomials of degree less than or equal to two. Determine $\{c_i\}_{i=0}^2$.

Question 2: [Eigenvalue]

Find the first two iterations obtained by the Power method and the inverse power method applied to

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix},$$

using $\mathbf{x}^{(0)} = (1, -1, 2)$.

Choose the linear functional

$$\varphi(\mathbf{x}) = \|\mathbf{x}\|_{\infty}.$$

Question 3: [Euler Method]

In a circuit with impressed voltage \mathcal{E} having resistance \mathcal{R} , inductance \mathcal{L} , and capacitance \mathcal{C} in parallel, the current i satisfies

$$\frac{di}{dt} = \mathcal{C} \frac{d^2 \mathcal{E}}{dt^2} + \frac{1}{\mathcal{R}} \frac{d\mathcal{E}}{dt} + \frac{\mathcal{E}}{\mathcal{L}}.$$

Suppose $\mathcal{C} = 0.3$ Farads, $\mathcal{R} = 1.4$ Ohms, $\mathcal{L} = 1.7$ Henrys, and

$$\mathcal{E}(t) = \sin(2t - \pi).$$

If $i(0) = 0$, approximate the current at $t_j = 0.1j$ for $j = 0, 1, 2$ using the Euler method.

Question 4: [Programming Exercise]

Write a Python code to implement the Trapezoidal rule to evaluate

$$\int_{-2}^2 x^2 e^x dx.$$

The exact value of the integral is

$$2e^2 - 10e^{-2} \approx 13.4226.$$

Divide the interval of integration into n equal subintervals and apply the Trapezoidal rule on each subinterval. Combine the results to obtain the final approximation.

Compute the numerical error for

$$n = 4, 8, 16.$$

Recall that if

$$a = x_0 < x_1 < \cdots < x_n = b,$$

then

$$\int_a^b f(x) dx = \sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}} f(x) dx.$$

