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MA203 Numerical Methods

Winter Semester 2026

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Assigment 5

This Tutorial is for practice. No need for submission.

Question 1: [Finite Difference Method]

The boundary value problem

$$y''(x) = 4\Big(y(x) - x\Big), \quad 0 \le x \le 1, \quad y(0) = 0, \quad y(1) = 2,$$

has the exact solution

$$y(x) = \frac{e^2}{e^4 - 1}\left(e^{2x} - e^{-2x}\right) + x.$$

Use the linear finite difference method with step size $h = 0.5$ to approximate the solution and compare the numerical result with the exact solution.

Question 2: [Elliptic Equation]

Using the five-point stencil, write the system of equations for the elliptic PDE

$$u_{xx} + u_{yy} = 4, \quad 0 < x < 1, \quad 0 < y < 2, \tag{1}$$

$$u(x,0) = x^2, \quad u(x,2) = (x-2)^2, \quad 0 \le x \le 1, \tag{2}$$

$$u(0,y) = y^2, \quad u(1,y) = (y-1)^2, \quad 0 \le y \le 2, \tag{3}$$

using $h = k = 0.5$.

Question 3: [Parabolic Equation]

Consider the one–dimensional heat equation

$$u_t = u_{xx}, \qquad 0 < x < \ell, \; t > 0,$$

subject to

$$u(0,t) = 0, \qquad u(\ell,t) = 0, \qquad t > 0,$$

and

$$u(x,0) = f(x), \qquad 0 \le x \le \ell.$$

Let $x_i = ih$ and $t_j = jk$.

(a) Using a forward difference at t_j and a backward difference at t_{j+1}, obtain two approximations for $u_t(x_i, t_{j+1/2})$.

(b) Take the average of the approximations obtained in part (a). Show that the resulting approximation is second–order accurate in time.

(c) Using central difference approximation for u_{xx} and the time approximation obtained in part (b), derive the fully discrete numerical scheme.